Robust Statistics for Signal Processing

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Motivation for Robust Statistics

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- Common assumptions such as Gaussianity, linearity and independence are only *approximations* to reality.

  Robust statistics is a body of knowledge, partly formalised into 'theories of robustness' relating to deviations from idealised assumptions in statistics [Hampel et al. (1986)].

- Robust statistics may be separated into two distinct but related areas
  - Robust Estimation - A robustification of classical estimation theory (point estimation)
  - Robust Testing - A robustification of the classical theory of statistical hypothesis testing (interval estimation)

Motivation for Robust Statistics (Cont'd)

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- Often engineering systems, such as in communication, are based on a parametric model where the observations are assumed to be Gaussian
- System performance is dependent on the accuracy of this model.
  - Classical parametric statistics are optimal under exact parametric models.
  - Performance becomes more uncertain the further we are from the assumed model.
  - Incorrect model leads to a performance decrease to an uncertain level.
- Systems designed using parametric models are very sensitive to deviations from the assumed model [Hampel *et al.* (1986), Huber (1981)].

Solution: Robust estimation of parametric models

Motivation for Robust Statistics (Cont'd)

- In general, the aims of robust statistics are to:
  - describe (or fit a model to) the majority of the data
  - identify (and deal with) outliers or influential points
- How far away from Gaussianity are we in reality? Experience shows [Hampel *et al.* 1986] that
  - high quality data sets may contain up to 1% outliers,
  - low quality data sets may contain more than 10% outliers,
  - $1 - 10\%$ outliers is common.

Example: Electricity consumption data

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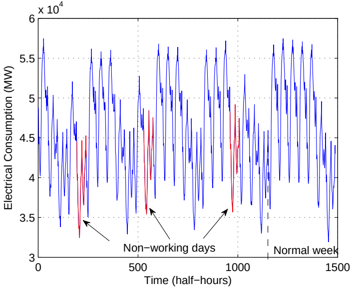
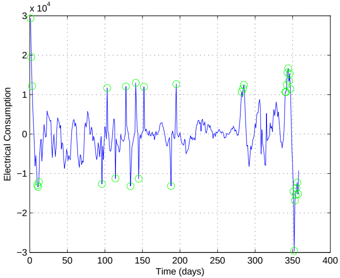


Half-hourly daily French electricity consumption on April 27^{th} to June 1^{st}, 2007



One-week differenced French load seasonal time series at 10:00, 2009

Motivation for Robust Statistics (Cont'd)

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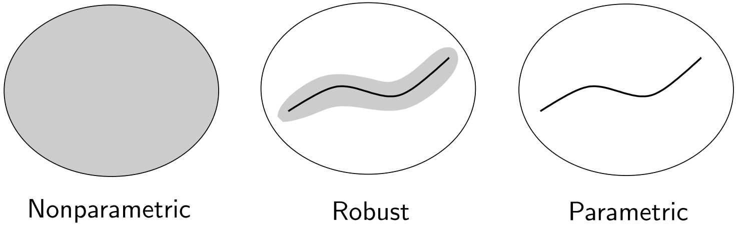
	Nonparametric	**Robust**	**Parametric**
Description	Model specified in terms of general properties	Parametric model allowing for deviations	Model completely specified by several parameters
Ideal Performance	Mediocre/Satisfactory	Good	Very Good/Excellent
Range of Validity	Large	Medium	Small



Robust is the most appropriate approach for real-life applications

Robust Estimation

Overview

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- Measures of Robustness: Quantitative and Qualitative robustness
- Location Estimation
- Linear Regression Models
- Correlated Data
- Signal Processing Applications

Quantitative Robustness

- Define the 'neighborhood' using e.g. the ε contaminated mixture model (or 'gross error model')

$$\mathcal{F} = \{F \mid F = (1-\varepsilon)F_0 + \varepsilon H\},$$

  where F_0 is the nominal distribution and H is the contaminating distribution.
- Consider
  1. maximum bias
  $$b(\varepsilon) = \sup_{F \in \mathcal{F}} |\Theta(F) - \Theta(F_0)|$$
  2. maximum variance
  $$v(\varepsilon) = \sup_{F \in \mathcal{F}} AV(F, \Theta)$$

Quantitative Robustness (Cont'd)

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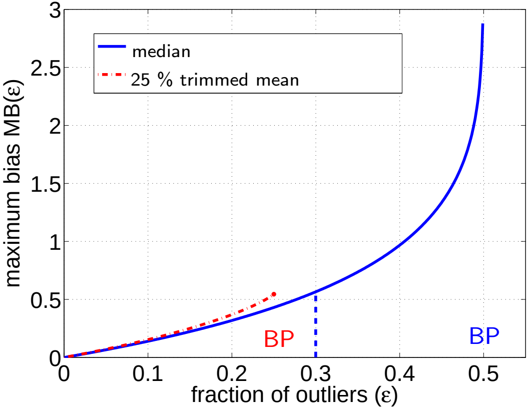
- Then the asymptotic breakdown point ε^* of an estimator at F_0 is

$$\varepsilon^*(F_0, \hat{\Theta}) = \sup\{\varepsilon | b(\varepsilon) < \infty\}.$$

  - Loosely speaking, it gives the limiting fraction of gross errors (outliers) the estimator can cope with (for details, see [Huber (1981) Section 1.4], [Hampel (1986) Section 2.2]).
  - In many cases ε^* does not depend on F_0 and is often the same for all the usual choices for $\mathcal{F}$.
- Maximum bias curve plots the maximum bias ($b(\varepsilon)$) of an estimator with respect to the fraction of contamination ε

Quantitative Robutsness: An Example



Maximum bias curves at $F_0 = \Phi$. Beyond the BP (ε^*), the maximum bias is infinite

Qualitative Robustness

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- The Influence function, introduced as influence curve [Hampel (1968,1974)], describes the effect (on an estimator $\hat{\Theta}$) of adding an observation of value x to a large sample; asymptotically, it is defined by

$$IF(x, F, \Theta) = \lim_{\Delta \to 0} \frac{\Theta((1-\Delta)F - \Delta\delta(x)) - \Theta(F)}{\Delta}$$

  where $\delta(x)$ denotes the point mass 1 at x.
- Roughly speaking, it is the first derivative of a statistic at F, where x plays the role of the contamination position.
- It measures the normalized asymptotic bias caused by an infinitesimal contamination at point x in the observations [Hampel (1986)].

Qualitative Robustness: An Example

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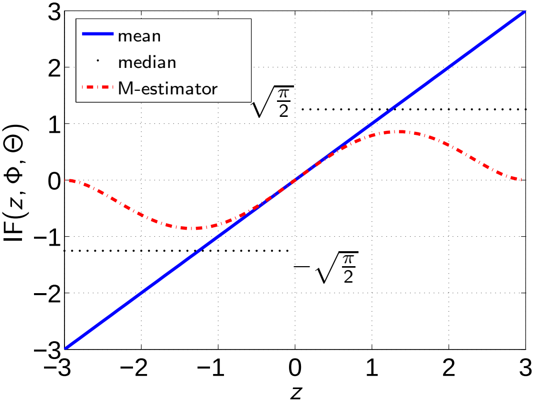

Influence functions of three estimators for the standard normal distribution Φ

Trade-off Robustness vs. Efficiency

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- Robustness: resistance of the estimator towards contamination quantified by: Breakdown Point ε^*, Maximum Bias Curve $b(\varepsilon)$, and Influence Function $IF(x, F, \Theta)$.
- Efficiency: the asymptotic behavior and variance of the estimator under the nominal model (clean data) quantified by $AV(F, \Theta)$ or $IF(x, F, \Theta)$.

$$AV(F, \Theta) = \int IF(x, F, \Theta)^2 dF(x)$$

Location Estimation

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Consider the model

$$X_t = \mu + \varepsilon_t$$

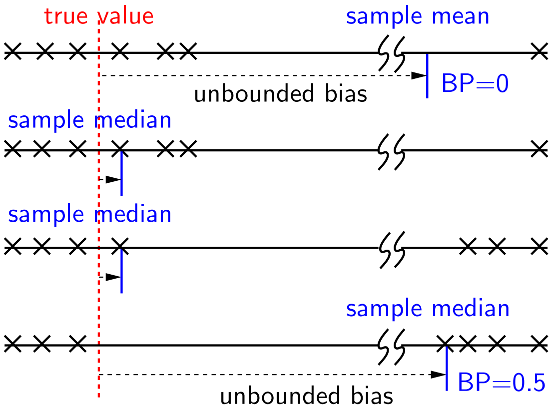
under a distribution F such that $X \sim F(x - \mu)$. We wish to estimate μ, given i.i.d $X_t,\ t = 1, \dots, n$. The Maximum likelihood estimate is

$$\widehat{\mu}_{ML} = \arg\max_{\mu} \sum_{t=1}^{n} \log f(x_t - \mu)$$

$$\Rightarrow \ \sum_{i=1}^{n} \psi(x_i - \widehat{\mu}_{ML}) = 0 \ \text{ where } \ \psi = f'/f$$

- F standard Gaussian: $\widehat{\mu}$ is the sample mean
- F double Exponential: $\widehat{\mu}$ is the sample median

Example of the Effect of Outliers



Effect of outliers on the bias of the sample mean and sample median

Example: Theoretical Robustness and Efficiency of the Sample Median

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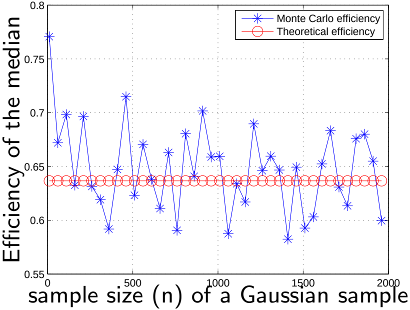
- IF(x,Φ,Med) bounded ⇒ sample median robust. Its BP $\varepsilon^*= 50$ %



- Median minimizes the maximum bias [Huber (1981)]. When $F_0 = \Phi$, efficiency of the Median is $2/\pi = 0.64$ ⇒ suggests M-estimators

M-Estimator

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- The MLE is asymptotically optimal (unbiased, consistent, achieves CRB) *only* if model is correct. If model is incorrect, performance uncertain $\rightarrow$ possibly not robust.
- Huber's approach generalises the MLE of a parameter θ of interest for independent observations by replacing $f_{X_t}(x_t|\theta)$ by an arbitrary function $\rho(x_t, \theta)$.

$$arg \max_{\theta} \sum_{t=1}^{N} \log f_{X_t}(x_t|\theta) \qquad \rightarrow \qquad arg \max_{\theta} \sum_{t=1}^{N} \rho(x_t, \theta)$$

- This estimator is called M-estimator (ML-type estimator).

M-Estimator (Cont'd)

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- Let ψ be the derivative of ρ, then

$$arg \max_{\theta} \sum_{t=1}^{N} \log f_{X_t}(x_t|\theta) \qquad \rightarrow \qquad arg \max_{\theta} \sum_{t=1}^{N} \rho(x_t, \theta)$$

leads to

$$\sum_{t=1}^{N} \frac{d \log f_{X_t}(x_t|\theta)}{d\theta} = 0 \qquad \rightarrow \qquad \sum_{t=1}^{N} \psi(x_t, \theta) = 0.$$

- ψ is called the score function since it 'scores' each observation x_t.

Robust M-estimators

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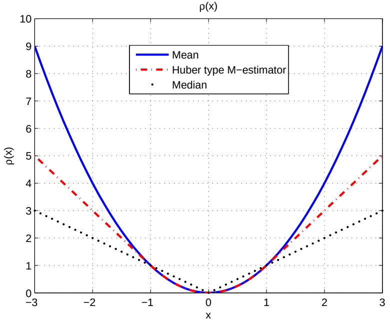
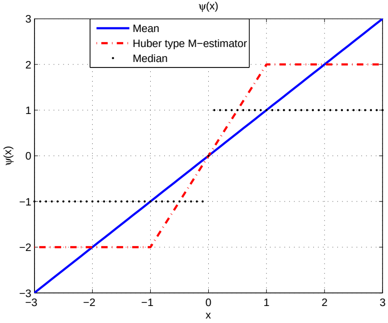
- The class of M-estimators contains in particular
  - The sample mean
  - The sample median
  - All maximum likelihood estimators
- Huber asked the following question
  How does one make an M-estimator robust?
- Implies determining f_X, or equivalently ψ, so that the M-estimator is robust
- An M-estimator is qualitatively robust if and only if ψ is bounded and continuous

Robust M-estimators



(a) ρ-function



(b) score function ψ

Huber M-estimator is qualitatively robust (ψ or IF bounded and continuous).

Linear Regression

Assume the model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$$

where $\mathbf{X}$ and $\mathbf{e}$ are independently distributed random variables. Let $\mathbf{Y}(t) = y_t$ and $\mathbf{x}_t'$ be the t^{th} row of the matrix $\mathbf{X}$.

Least-Squares Estimator (LSE)

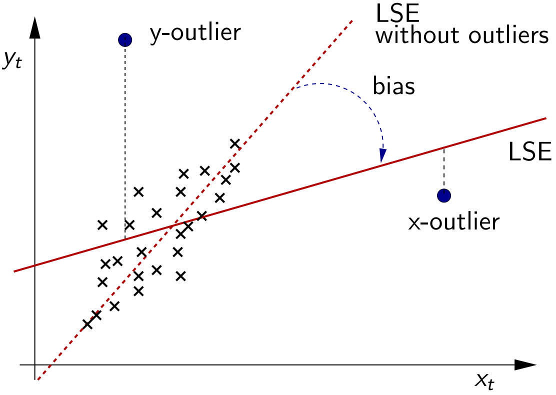
$$\widehat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \sum_{t=1}^{n} r_t(\boldsymbol{\beta})^2$$

$r_t(\boldsymbol{\beta}) = y_t - \mathbf{x}_t'\boldsymbol{\beta}$, being the t^{th} residual. The LSE solves for

$$\sum_{t=1}^{n} r_t(\widehat{\boldsymbol{\beta}})\mathbf{x}_t = \mathbf{0}$$

Effect of Outliers

The regression line is tilted by the outliers



M-Estimation for Regression

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M-Estimator:

$$\widehat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \sum_{t=1}^{n} \rho\left(\frac{r_t(\boldsymbol{\beta})}{\widehat{\sigma}_r}\right) \Rightarrow \sum_{t=1}^{n} \psi\left(\frac{r_t(\widehat{\boldsymbol{\beta}})}{\widehat{\sigma}_r}\right) \mathbf{x}_t = \mathbf{0}$$

A leverage point ($\|x_t\|$ large) will dominate the equation. Here, $\widehat{\sigma}_r$ is a robust scale of the residuals r_t.

Solution:

- Redescending function ψ: MM estimator
- Down-weight leverage points: GM estimator
- Other robust estimators are: Residual Autocovariance (RA-), Least Median Squares (LMS), Least Trimmed Squares (LTS), S-, τ-Estimators.

Leverage Points in Regression

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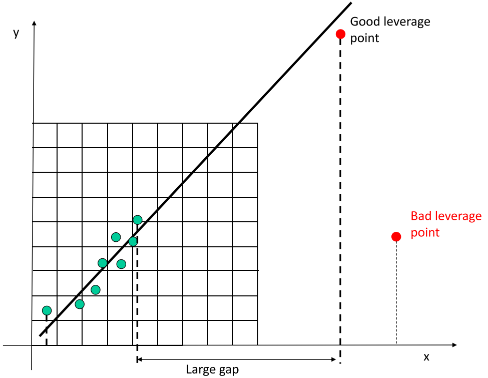
LSE and M-Estimator are not qualitatively robust



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Robust Methods for Correlated Data

- Difficulty introduced by the correlation
- Several types of outliers: Additive, Innovative, patchy, isolated, ...
- Different definitions of robustness measures
- Estimators need to be adapted to deal with correlation

$$\Downarrow$$

Only a few methods exist

- 2 Definitions of influence function in the correlated data case ([Künsch (1981)] and [Martin (1981)])
- Breakdown Point (ε^*) still not clearly defined [Genton (2010)]

Some Robust Estimators for ARMA Models

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- CML: Cleaned Maximum Likelihood after '3-σ' rejection (practical engineering method) $\Rightarrow$ it neglects the correlation, bad performance.
- Generalized-M (GM): used in Power Systems [Maronna *et al.* (2006)], not robust for ARMA; for AR(p), BP (ε^*) decreases with increasing p.
- Residual Autocovariance (RA) and truncated RA estimators (TRA) [Bustos and Yohai (1986)]. RA is not robust and TRA lacks efficiency for ARMA.
- Filtered-M: M-estimator combined with robust filter [Maronna (2006)]. Robust but lacks efficiency.
- Filtered-τ [Maronna (2006)], Ratios of Medians (RME), Medians of Ratios (MRE) and Filtered Hellinger based estimator [Chakhchoukh (2010)].

Importance of Robust Filtering

For an AR(p), a residual r_t is evaluated by regressing y_t on the p variables $y_{t-1}, \dots, y_{t-p}$. An observation y_t is used in computing $p+1$ residuals: $r_t, \dots, r_{t+p}$
$\Rightarrow$ BP (ε^*) decreases significantly.

For an ARMA model, one observation will affect all the residuals $\Rightarrow$ BP $\varepsilon^* = 0$.

Problem: Propagation of outliers in the data used in the estimation
Solution: use robust residuals computed by a robust filter cleaner [Masreliez (2010)]:

$$\widetilde{r}_t = y_t - \phi_1 \widehat{y}_{t-1|t-1} - \dots - \phi_p \widehat{y}_{t-p|t-1}$$

Cleaning the data with a robust filter improves efficiency

Robust Filtering of an AR(1)

For an AR(1): $X_t = \phi X_{t-1} + \varepsilon_t$, we use the following robust filtering algorithm:

- estimate robustly $\widehat{\phi}$, e.g.: $\widehat{\phi} = \widehat{C}_r(1)/\widehat{C}_r(0)$ and run the recursions of the filter-cleaner; $\widehat{X}_{1|1} = X_1$, $P_{1|1} = MADN(X_t)$,

Prediction :

$$\widehat{X}_{2|1} = \widehat{\phi}\widehat{X}_{1|1};$$
$$\widehat{\varepsilon}_2 = Y_2 - \widehat{\phi}\widehat{X}_{1|1}$$
$$P_{2|1} = \widehat{\phi}2P_{1|1} + \sigma_\varepsilon 2$$

Correction :

$$\widehat{X}_{2|2} = \widehat{X}_{2|1} + \frac{1}{\sqrt{P_{2|1}}} P_{2|1}\psi\left(\frac{\widehat{\varepsilon}_2}{\sqrt{P_{2|1}}}\right)$$
$$P_{2|2} = P_{2|1} - \frac{1}{P_{2|1}} P_{2|1}^2 w\left(\frac{\widehat{\varepsilon}_2}{\sqrt{P_{2|1}}}\right)$$

Go to the next step of the recursion.

- Apply the ML to the filtered series $\{\widehat{X}_{t|t}\}$ or:
- Test to remove the outliers, e.g.: If $\widehat{\varepsilon}_2 > 3\sqrt{P_{2|1}}$ then X_2 is outlying
- Apply ML estimator that handles missing data.

Applications

- Wireless Communications: Robust Geolocation, eg: [Hammes (2009)]
- Array Processing: Robust Direction of Arrival Estimation, eg: [Tsakalides (1995)]

Wireless Communications: Robust geolocation

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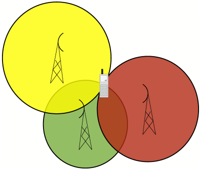
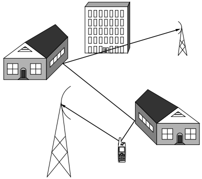
- Geolocation refers to identifying the position of a mobile terminal using a network of sensors.
- Applications for Geolocation arise e.g. in emergency call services, yellow page services and intelligent transport systems [Caffery (1999)].
- We consider wireless positioning of a stationary terminal based on TOA estimates.
- At least three sensors/BSs are needed to solve ambiguities



line-of-sight (LOS)



non-line-of-sight (NLOS)

Problem Statement

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- Time of arrival (TOA) estimates are multiplied by the speed of light to obtain the measured distances

$$r_m = \underbrace{\sqrt{(x_m - x)^2 + (y - y_m)^2}}_{=h_m} + \tilde{v}_m, \qquad m = 1, \dots, M,$$

where x_m, y_m are the known coordinates of the BS and x, y describe the unknown location of the MT. The i.i.d. random variables $\tilde{v}_m$ have pdf

$$f_{\tilde{V}}(\tilde{v}) = (1-\varepsilon)\mathcal{N}(\tilde{v}; 0, \sigma_G) + \varepsilon\mathcal{H},$$

describing sensor noise and errors due to NLOS propagation ($\mathcal{H} = f_G * f_\eta$) where f_η may be any pdf with positive mean such that $\mathrm{E}\{\mathcal{H}\} > 0$.

Linearization

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- Squaring the nonlinear equation yields

$$r_m^2 = h_m^2 + \underbrace{2h_m\tilde{v}_m + \tilde{v}_m^2}_{=v_m(h_m)} \qquad m = 1, \dots, M$$

- For M BSs we have

$$\mathbf{r} = \mathbf{S}\boldsymbol{\theta} + \mathbf{v},$$

  where $\boldsymbol{\theta} = [x\ y\ R^2]^\mathsf{T}$ with $R^2 = x^2 + y^2$.
- Since $f_V(v)$ is non-Gaussian and contains outliers due to NLOS, least-squares estimation suffers from a performance loss.

$$\Downarrow$$

Robust methods

Iterative Robust Algorithm

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1. **Initialisation:** Set $i = 0$. Obtain an initial estimate of $\boldsymbol{\theta}$, $\hat{\boldsymbol{\theta}}^0$.

2. **Determine residuals:** $\hat{\mathbf{v}} = \mathbf{r} - \mathbf{S}\hat{\boldsymbol{\theta}}^i$.

3. **Estimate λ, perform transformation KDE.**

4. **Estimate score function:** $\hat{\varphi} = -\frac{\hat{f}'_V(v)}{\hat{f}_V(v)}$.

5. **Update:** $\hat{\boldsymbol{\theta}}^{i+1} = \boxed{\hat{\boldsymbol{\theta}}^i + \mu(\mathbf{S}^\top\mathbf{S})^{-1}\mathbf{S}^\top\hat{\varphi}(\hat{\mathbf{v}})}$ or $\boxed{(\mathbf{S}^\top\boldsymbol{\Omega}\mathbf{S})^{-1}\mathbf{S}^\top\boldsymbol{\Omega}\mathbf{r},\ \boldsymbol{\Omega} = \mathrm{diag}(\boldsymbol{\omega}),\ \boldsymbol{\omega} = |\hat{\varphi}(\hat{\mathbf{v}})/\hat{\mathbf{v}}|}$

6. **Check for convergence:** If $\frac{\|\hat{\boldsymbol{\theta}}_{i+1} - \hat{\boldsymbol{\theta}}_i\|}{\|\hat{\boldsymbol{\theta}}_{i+1}\|} < \xi$ stop, otherwise set $i \to i+1$ and go to step 2.

Simulation Settings

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- Consider 10 BSs each of them collecting 10 measurements.
- We compare least-squares ('LS') with Huber's M-estimator ('H_c') where the clipping point $c = 0.6\hat{\sigma}_V$, where $\hat{\sigma}_V$ is estimated using the median absolute deviation.
- Semi-parametric estimators using Newton-Raphson algorithm labeled as 'SPMR' and the one based on weighted least-squares is 'SPWLS'.
- We average over $MC = 10,000$ Monte-Carlo runs and $\sigma_G = 150m$.
- Performance measure is the mean error distance, i.e.,

$$MED = \frac{1}{MC}\sum_{i=1}^{MC}\sqrt{(x-\hat{x}_i)^2 + (y-\hat{y}_i)^2}$$

Simulation results

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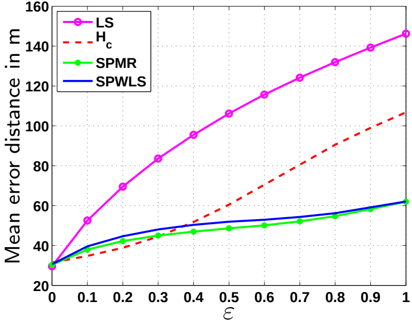

MED vs. degree of NLOS contamination. f_η is an exponential distribution with $\sigma_\eta = 409m$

Array Processing: Robust Direction of Arrival Estimation

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- Direction of Arrival (DOA) estimates are needed in array processing
  - Smart Antennas
  - Space-Time Adaptive Processing
  - Radar
- Classical methods for DOA estimation based on sample covariance matrix are not robust
  - Beamformer
  - Capon's minimum variance
  - ML techniques
  - Subspace methods MUSIC, ESPIRIT

Array Processing: Robust Direction of Arrival Estimation

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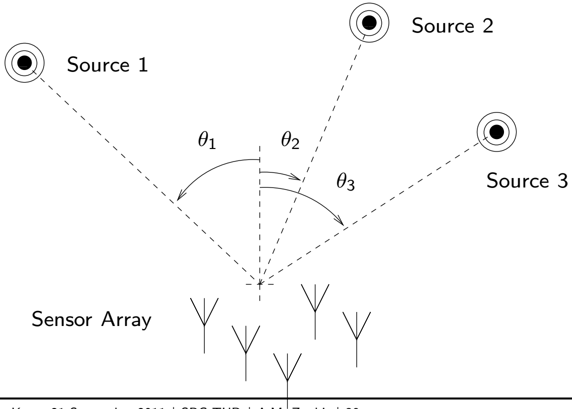
- Robust DOA methods exist based on
  - M-Estimation
  - Symmetric-Alpha-Stable (SaS) distributions and Fractional Lower Order Moments (FLOMs)
  - Gaussian mixture distributions and Space Alternating Generalised Expectation Maximisation (SAGE)
  - Nonparametric statistics using the spatial sign function
- Former three robust DOA estimators require knowledge of noise parameters/setting of thresholds/choice of weighting functions
- Last nonparametric estimator is simple and requires no prior knowledge or settings to be chosen

Array Processing: Robust Direction of Arrival Estimation

Problem: Estimate the direction-of-arrivals of the sources using the observations of a sensor array in an impulsive noise environment



Array Signal Model

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Model

$$\mathbf{y}_n = \mathbf{A}\mathbf{s}_n + \mathbf{x}_n, \qquad n = 1, \dots, N$$

$\mathbf{y}_n$: p-dim snapshot from p array elements

$\mathbf{A} = (\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_q))$: $p \times q$-dim array steering matrix

$\mathbf{a}(\theta)$: p-dim array steering vector

$\theta_1, \dots, \theta_q$: directions to the q sources

$\mathbf{s}_n$: q-dim element source signal

$\mathbf{x}_n$: p-dim spherically symmetric noise

Assumptions

- snapshots $\mathbf{y}_n$, $n = 1, \dots, N$, are i.i.d.
- source signal $\mathbf{s}_n$ and noise $\mathbf{x}_m$ are independent for all $n, m = 1, \dots, N$
- $q < p$ to avoid identifiability problems
- $\mathbf{A}$ is of full rank q

Conventional DOA estimation: MUSIC

The spatial covariance matrix has the following structure

$$\mathbf{R} = \mathbf{A}\ \mathbf{R}_s \mathbf{A}^H + \sigma^2 \mathbf{I} \qquad (1)$$

From the eigendecomposition of R

$$\mathbf{R} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V} \qquad (2)$$

where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_M]$ and $\boldsymbol{\Sigma} = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_M]$.

- Construct the signal and the noise subspace, $\mathbf{U}_s$ and $\mathbf{U}_n$, respectively
- Search for the peaks in the MUSIC pseudo-Spectrum $P(\theta)$

$$P(\theta) = \frac{1}{\|\mathbf{U}_n^H \mathbf{a}(\theta)\|^2} \qquad (3)$$

Conventional DOA Estimation

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The estimated covariance matrix is given by

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}\mathbf{x}^H$$

$$\Downarrow$$

Robust Estimation:

- Normalize each of the snapshots, called spatial sign function
- Trim the corrupted observations, requires hypothesis testing
- Robust estimation of spatial covariance matrix, e.g. FLOM, maximum likelihood estimation. Some possible methods for robust covariance estimation MCD, MVE, MM-, ...

Spatial Sign Function & Robust Covariance

Spatial Sign Function

- Spatial sign function (SSF) of a *p*-variate complex vector **x**

$$\mathbf{u}(\mathbf{x}) = \begin{cases} \frac{x}{||x||} & \mathbf{x} \neq 0 \\ \mathbf{0} & \mathbf{x} = 0 \end{cases}$$

- **u** is a unit length direction vector
- Generalises the sign function $sgn(x)$ for 1-D to *p*-D

Spatial Sign Function & Robust Covariance

- Sample spatial sign covariance matrix (SCM) of

$$R_1 = E[\mathbf{u}(\mathbf{x})\mathbf{u}^H(\mathbf{x})]$$

$$\hat{R}_1 = \frac{1}{N}\sum_{n=1}^{N}\mathbf{u}(\mathbf{x}_n)\mathbf{u}^H(\mathbf{x}_n)$$

also known as quadrant correlation

- Sample spatial tau covariance matrix (TCM) of

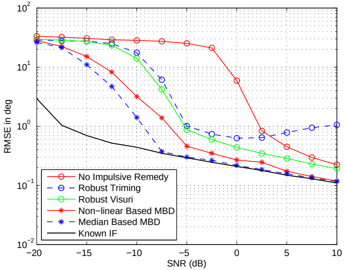
$$R_2 = E[\mathbf{u}(\mathbf{x}-\mathbf{y})\mathbf{u}^H(\mathbf{x}-\mathbf{y})]$$

$$\hat{R}_2 = \frac{1}{N(N-1)}\sum_{n=1}^{N}\sum_{m=1}^{N}\mathbf{u}(\mathbf{x}_n-\mathbf{x}_m)\mathbf{u}^H(\mathbf{x}_n-\mathbf{x}_m)$$

Simulation results

Setup

- Two linear FM signals impinging on an array of $m = 8$ sensors in ULA geometry
- DOAs are $[-3^o \ 2^o]$
- Total number of snapshots $N = 128$
- ϵ-contaminated mixture, $\epsilon = 0.2$ and $\kappa = 20$



RMSE DOA Estimation

Conclusions

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- Robust methods are useful tools for estimation in many real world applications.
- Robust statistics for independently and identically distributed data are well-established.
- There exists a need for robust techniques for correlated data: the more interesting case for a signal processing practitioner.
- Optimality has its advantage, but *Robustness* is the engineer's interest.

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